# List of definitions for the Final

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This list is very complete! It includes **all** the definitions you need to know for the final!

## 1 $\lim_{x\to a} f(x) = L$ (limit of $f(x)$ as $x$ approaches $a$ is $L$)

Let $f$ be a function defined on some open interval that contains the number $a$, except possibly at $a$ itself. Then we say that the **limit of** $f(x)$ **as** $x$ **approaches** $a$ **is** $L$, and we write

$$\lim_{x\to a} f(x) = L$$

if for every number $\epsilon > 0$ there is a number $\delta > 0$ such that

$$\text{if} \qquad 0 < |x-a| < \delta \qquad \text{then} \qquad |f(x) - L| < \epsilon$$

## 2 $f$ continuous at $a$

A function $f$ is **continuous at a number a** if

$$\lim_{x\to a} f(x) = f(a)$$

## 3 $f'(a)$ (derivative of $f$ at $a$)

The **derivative of a function** $f$ **at a number** $a$, denoted by $f'(a)$ is

$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

**if this limit exists**

## 4 $f$ differentiable on $a$, $f$ differentiable on $(a, b)$

A function $f$ is **differentiable at** $a$ if $f'(a)$ exists. It is **differentiable on an open interval** $(a, b)$ [or $(a,\infty)$ or $(-\infty, a)$ or $(-\infty,\infty)$ if it is differentiable at every number in the interval.

## 5 The Extreme Value Theorem

**Extreme Value Theorem:** If $f$ is continuous on a closed interval $[a, b]$ then $f$ attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers $c$ and $d$ in $[a, b]$.

## 6 Mean Value Theorem

**Mean Value Theorem:** Let $f$ be a function that satisfies the following two hypotheses:

1) $f$ is continuous on the closed interval $[a, b]$
2) $f$ is differentiable on the open interval $(a, b)$

Then there is a number $c$ in $(a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a)$$

## 7 The definite integral

If $f$ is a function defined for $a \leq x \leq b$, we divide the interval $[a, b]$ into $n$ subintervals of equal width $\Delta x = \frac{b-a}{n}$. We let $x_0(= a), x_1, x_2, \cdots, x_n(= b)$ be the endpoints of these subintervals, and we let $x_1^*, x_2^*, \cdots, x_n^*$ be any **sample points** in these subintervals, so $x_i^*$ lies in the ith subinterval $[x_{i-1}, x_i]$. Then **the definite integral of** $f$ **from** $a$ **to** $b$ is:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

provided that this limit exists. If it does exist, we say that $f$ is **integrable** on $[a, b]$

## 8 Continuous functions are integrable

If $f$ is continuous on $[a, b]$, or if $f$ has only a finite number of jump discontinuities, then $f$ is integrable on $[a, b]$; that is, the definite integral $\int_a^b f(x)dx$ exists.

# 9 The Fundamental Theorem of Calculus - Part I

If $f$ is continuous on $[a, b]$, then the function $g$ defined by

$$g(x) = \int_a^x f(t)dt \qquad a \leq x \leq b$$

is continuous on $[a, b]$, and differentiable on $(a, b)$, and $g'(x) = f(x)$

# 10 The Fundamental Theorem of Calculus - Part II

If $f$ is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

where $F$ is any antiderivative of $f$, that is, a function such that $F' = f$

# 11 Volume

Let $S$ be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area $S$ in the plane $P_x$, through $x$ and perpendicular to the $x-$axis is $A(x)$, where $A$ is a continuous function, then the **volume** of $S$ is:

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i^*)\Delta x = \int_a^b A(x)dx$$

## Test yourself!

Now, without looking at the definitions on the previous pages, try to define the following terms. Then compare your answers to the definitions above, and correct any mistake you make. You have to memorize those definitions **word by word**, e-mail me if you have any doubts about a definition!

1. $\int_a^b f(x)dx$ (the definite integral of $f$ from $a$ to $b$)
2. Derivative of $f$ at $a$
3. The fundamental theorem of Calculus - Part II
4. $f$ differentiable at $a$, $f$ differentiable on $(a, b)$
5. $f$ continuous at $a$
6. The Mean Value Theorem
7. Volume
8. The Extreme Value Theorem
9. The fundamental theorem of Calculus - Part I
10. Continuous functions are integrable
11. $\lim_{x\to a} f(x) = L$